The complexity of optimizing over strictly local constraints

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PLC 43
Spring 2019

Introduction

- In Optimality Theory (Prince & Smolensky 1993), the interaction of local constraints can produce non-local, pathological patterns
- Use categories of patterns provided by Formal Language Theory (FLT) to contrast attested patterns with unattested ones
- Analyze a typology of stress constraints, but property of OT grammars is general - potentially true of *any* set of local constraints

Introduction

- Pathological pattern is novel "sour grapes"-like stress pattern from local markedness constraints only

  σ
  $(\acute{\sigma}\sigma)$
  $\sigma\sigma\sigma$
  $(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$
  $\sigma\sigma\sigma\sigma\sigma$
  $(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$
  $\sigma\sigma\sigma\sigma\sigma\sigma\sigma$
  $\ldots$

- Tells us that restricting CON in some way is no guarantee of a typology with matching complexity

Plan

- Background
- Introduce the constraint set
- Explore the pattern in detail
- Show how and why the pattern is pathological
- Discuss implications

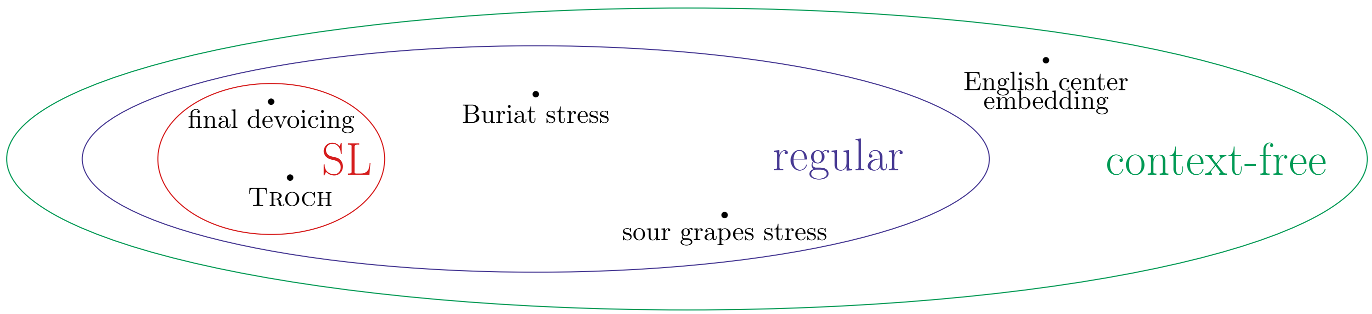
FLT

- Formal languages describe *stringsets* that are extensions of the grammar, ex. $^*ab = \{a, aa, bb, ba, baa, ...\}$
- Can think of constraints this way as well: $\textsc{Troch} = \{(\acute{\sigma}\sigma), (\acute{\sigma}\sigma)\sigma, ...\}$
- Phonological patterns: "final devoicing" informally describes set of strings that are well formed with regard to the generalization of the pattern

Measuring Complexity

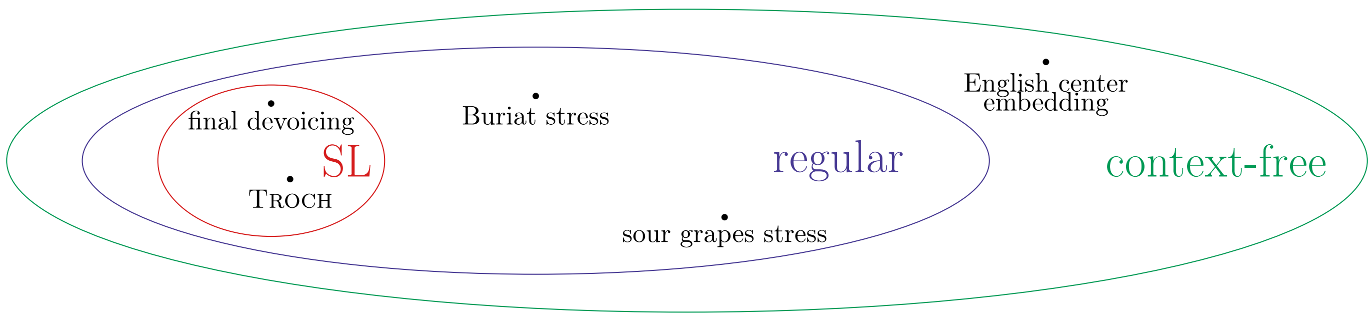
- Can use principles of formal language theory to measure *complexity* of natural language patterns
- What kind of FLT grammar describes a phonological pattern? A local one like $^*ab$? Something more powerful?
- Gives us rigorously-defined notion of what a possible phonological generalization is

Measuring Complexity



- Chomsky Hierarchy of formal langauges; division of space of possible grammars based on expressive power of those grammars
- Phonology is *regular* (Rogers et al. 2013; Heinz 2018): expect phonological patterns to fall within the blue region
- Something intuitively non-phonological about center embedding, FLT tells us exactly why

Measuring Complexity



- Most phonological patterns are *sub-regular* (Heinz 2018), part of some even more restricted class

- *Strictly Local* (SL) class (McNaughton & Papert 1971; Rogers & Pullum 2011) at very bottom, formalize what we mean by "local"

Strictly Local

- SL class definable with *conjunctions of negative literals* (CNLs), where literals are substructure: $\neg\ s_1 \wedge \neg\ s_2 \wedge \ldots\ s_n$
- Statements forbidding contiguous substructures, no requirement of structure
- Relevant to markedness constraints in OT, overwhelmingly negative i.e. ban certain structures
- Example: TROCH, bans iambs and unary feet $\neg\ (\sigma\acute{\sigma}) \wedge \neg(\acute{\sigma})$

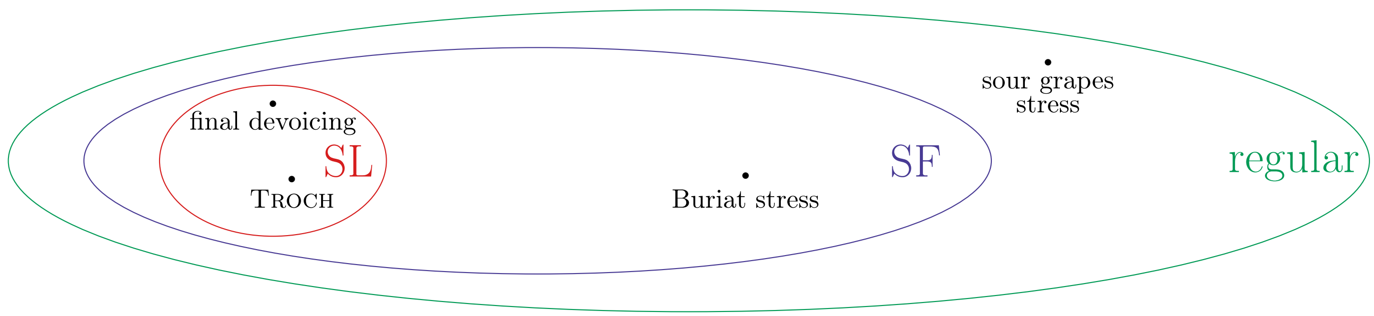
SL Constraints

- Will define constraints as strictly local
- Use SL as the *Constraint Definition Language* (CDL) (Eisner 1997; de Lacy 2011; Jardine & Heinz 2016) for stress markedness constraints
- Strong prediction that markedness constraints forbid local structures only
- Cannot write constraints of a higher complexity
  ex. FirstAndLast - "stress the last syllable if the first syllable is stressed"

SL Constraint Interaction

- McNaughton & Papert (1971): SL stringsets *closed* under intersection: intersection of two SL stringsets is guaranteed to result in SL stringset
- no jump to higher level of complexity
- can ask same question of optimization in OT:
    - if optimization is how constraints (stringsets) interact, is there the same kind of complexity class closure?
- No.

Other Relevant Classes



- Natural language stress patterns are overwhelmingly *star free* (SF) (Rogers et al. 2013)
- Sour grapes pattern examined here is not - it is fully regular
- Again, FLT provides explanation as to why pattern seems unnatural – let's see how it arises

Gen

- Consider strings of syllables – unstressed σ, stressed $\acute{\sigma}$, unparsed $\breve{\sigma}$, and foot boundaries right), and left (
- $(\acute{\sigma}\sigma)\breve{\sigma}\breve{\sigma}\breve{\sigma}$ or $(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$
- No superbinary feet (this requirement is SL)
- Allow stressless strings; obligatoriness (requiring at least one stress) Locally Testable; Rogers et al. (2013)

Constraints

- Defined with CNL logic
- Count number of violations – number of ill-formed structures
- TROCH: $\neg\ (\sigma\acute{\sigma}) \wedge \neg\ (\acute{\sigma})$
  - Violated by strings $\breve{\sigma}(\sigma\acute{\sigma})$ and $(\acute{\sigma})(\sigma\acute{\sigma})$
  - Unviolated by strings $\breve{\sigma}\breve{\sigma}(\acute{\sigma}\sigma)$ and $(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$
- Defined over alphabet $\Sigma = \{(,), \sigma, \acute{\sigma}, \breve{\sigma}\}$

Constraints

Constraint set:

IAMB violated by trochees and unary feet; $\neg\ (\acute{\sigma}\sigma) \wedge \neg\ (\acute{\sigma})$
TROCHEE violated by iambs and unary feet; $\neg\ (\sigma\acute{\sigma}) \wedge \neg\ (\acute{\sigma})$
PARSE violated by an unparsed syllable; $\neg\ \breve{\sigma}$
$^{*}\breve{\sigma}F$; $\neg\breve{\sigma}(\sigma \wedge \neg\breve{\sigma}(\acute{\sigma}$
$^{*}F\breve{\sigma}$; $\neg\sigma)\breve{\sigma} \wedge \neg\acute{\sigma})\breve{\sigma}$

- Basic stress constraints needed for a local theory of CON for stress
- All constraints from the literature with an explicit CNL definition
- Application of constraints consistent with use in literature

Constraints

- $^*\breve{\sigma}F$ and $^*F\breve{\sigma}$
  $\neg\breve{\sigma}(\sigma \wedge \neg\breve{\sigma}(\acute{\sigma}$ and $\neg\sigma)\breve{\sigma} \wedge \neg\acute{\sigma})\breve{\sigma}$

$^*\breve{\sigma}F$		$^*F\breve{\sigma}$	
$(\acute{\sigma}\sigma)\breve{\sigma}(\acute{\sigma}\sigma)$	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$	$(\acute{\sigma}\sigma)\breve{\sigma}(\acute{\sigma}\sigma)$	$\breve{\sigma}(\acute{\sigma}\sigma)$
$\breve{\sigma}(\acute{\sigma}\sigma)\breve{\sigma}(\acute{\sigma}\sigma)$	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)\breve{\sigma}$	$(\acute{\sigma}\sigma)\breve{\sigma}$	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$

- Motivate placement of feet
- Similar to *Ft/_σ and *Ft/σ_ discussed in McCarthy (2003); defined as CNLs

Constraints

- Troch and Iamb
  $\neg\ (\sigma\acute{\sigma}) \wedge \neg\ (\acute{\sigma})$ and $\neg\ (\acute{\sigma}\sigma) \wedge \neg\ (\acute{\sigma})$

Troch		Iamb	
$(\acute{\sigma}\sigma)(\sigma\acute{\sigma})$	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$	$(\acute{\sigma}\sigma)(\sigma\acute{\sigma})$	$\breve{\sigma}(\sigma\acute{\sigma})$
$(\sigma\acute{\sigma})(\acute{\sigma})$	$(\acute{\sigma}\sigma)$	$(\acute{\sigma}\sigma)(\acute{\sigma})$	$(\sigma\acute{\sigma})(\sigma\acute{\sigma})$

Constraints

- PARSE: constraint against unparsed syllables
  $\neg\ \breve{\sigma}$

PARSE	
$(\acute{\sigma}\sigma)\breve{\sigma}$	$(\acute{\sigma}\sigma)$
$(\acute{\sigma}\sigma)\breve{\sigma}\breve{\sigma}$	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$
$(\acute{\sigma}\sigma)\breve{\sigma}\breve{\sigma}\breve{\sigma}$	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)(\acute{\sigma})$

Typology

- Analysis in OTWorkplace (Prince et al. 2007-2017) reveals typology of 9 languages: 2 sour grapes languages, 1 stress-less language, 2 ambiguous languages (more than one optimal output), 4 near-misses of attested patterns (iterating binary feet)

Sour Grapes Stress

- Sour grapes is a pathology in harmony generated by some theories of assimilation in OT (Padgett 1995; Wilson 2003, 2006; McCarthy 2010)

	sour grapes harmony:		natural language harmony:
*	+F -F -F -F -F	*	+F -F -F -F -F
	+F +F +F +F +F		+F +F +F +F +F
	+F -F -F B_F -F		+F +F +F B_F -F

- If some feature cannot spread completely, candidate with no spreading wins instead

Sour Grapes Stress

- Can generate similar pattern in stress with only SL markedness constraints

$\breve{\sigma}$

$(\acute{\sigma}\sigma)$

$\breve{\sigma}\breve{\sigma}\breve{\sigma}$

$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$

$\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}$

$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$

$\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}$

...

- Pathological – no such extreme sensitivity to word length in natural language stress patterns

Sour Grapes Stress

$\breve{\sigma}$
$(\acute{\sigma}\sigma)$
$\breve{\sigma}\breve{\sigma}\breve{\sigma}$
$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$
$\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}$
$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$
$\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}$
...

input	winner	loser	$*\sigma F$	$*F\sigma$	TROCH	PARSE	IAMB
3syll	$\breve{\sigma}\breve{\sigma}\breve{\sigma}$	$\breve{\sigma}(\acute{\sigma}\sigma)$	W			L	W
3syll	$\breve{\sigma}\breve{\sigma}\breve{\sigma}$	$(\acute{\sigma}\sigma)\breve{\sigma}$		W		L	W
3syll	$\breve{\sigma}\breve{\sigma}\breve{\sigma}$	$(\acute{\sigma}\sigma)(\acute{\sigma})$			W	L	W
4syll	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$	$\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}$				W	L
4syll	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$	$(\acute{\sigma}\sigma)\breve{\sigma}\breve{\sigma}$	W			W	L

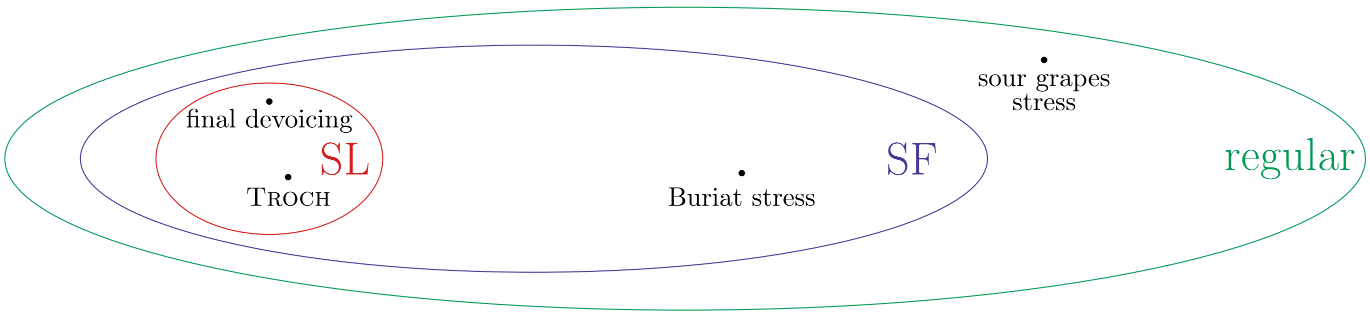
Sour Grapes Stress

$\breve{\sigma}$
$(\acute{\sigma}\sigma)$
$\breve{\sigma}\breve{\sigma}\breve{\sigma}$
$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$
$\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}$
$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$
$\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}$
...

input	winner	loser	$*\breve{\sigma}F$	$*F\breve{\sigma}$	TROCH	PARSE	IAMB
3syll	$\breve{\sigma}\breve{\sigma}\breve{\sigma}$	$\breve{\sigma}(\acute{\sigma}\sigma)$	W			L	W
3syll	$\breve{\sigma}\breve{\sigma}\breve{\sigma}$	$(\acute{\sigma}\sigma)\breve{\sigma}$		W		L	W
3syll	$\breve{\sigma}\breve{\sigma}\breve{\sigma}$	$(\acute{\sigma}\sigma)(\acute{\sigma})$			W	L	W
4syll	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$	$\breve{\sigma}\breve{\sigma}\breve{\sigma}\breve{\sigma}$				W	L
4syll	$(\acute{\sigma}\sigma)(\acute{\sigma}\sigma)$	$(\acute{\sigma}\sigma)\breve{\sigma}\breve{\sigma}$	W			W	L

- In odd-syllable forms, cannot satisfy $*\breve{\sigma}F$ or $*F\breve{\sigma}$ with binary feet
- Any unary feet violate TROCH
- In even syllable forms, full satisfaction of PARSE – anything less incurs violations of higher ranked constraints

Sour Grapes Stress



- Sour grapes-like stress pattern from markedness constraints only
- Arises from interaction of SL constraints, pattern is properly regular
- SL class is not closed under optimization

Not Star Free

- Sour grapes pattern discussed here is regular (see Appendix)
- Can also show is not star free, and thus unlike natural language stress patterns
- Alphabet change: $\Sigma = \{(,), \sigma\}$
- Sour grapes-like language as a stringset:

$$\begin{aligned} L = \ & \{\sigma, \\ & (\sigma\sigma), \\ & \sigma\sigma\sigma, \\ & (\sigma\sigma)(\sigma\sigma), \\ & \sigma\sigma\sigma\sigma\sigma, \\ & (\sigma\sigma)(\sigma\sigma)(\sigma\sigma), \\ & \sigma\sigma\sigma\sigma\sigma\sigma\sigma, \ldots\} \end{aligned}$$

Not Star Free

- **Theorem 1** (McNaughton & Papert 1971)
  - $\exists n$ such that $\forall i$ $uv^n w \in L \rightarrow uv^{n+i} w \in L$
- No string $\sigma\sigma\sigma^n$ for even n, can use as target for uv^{n+i}w

Odd n, $i = 1$, $v = \sigma$

n	$uv^n w \in L$	$\rightarrow$	$uv^{n+i} w \in L$
1	$\sigma\sigma\sigma\sigma\sigma$		$\sigma\sigma\sigma\sigma\sigma\sigma \notin L$
3	$\sigma\sigma\sigma\sigma\sigma\sigma\sigma$		$\sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma \notin L$
5	$\sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma$		$\sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma \notin L$
		$\vdots$	

- Can construct same argument for even n (see Appendix)

Not Star Free

- It is not the case that for all $i \geq 1$, there is an odd n or even n such that if $uv^{\mathrm{n}}w$ is a string of L then $uv^{\mathrm{n+i}}w$ is a string of L for all $i \geq 1$
- Proves that Thm. 1 does not hold for the sour grapes-style pattern and thus that it is not SF

Discussion

- A system of SL constraints that produced a fully regular pattern via interaction in OT
- Pattern was a novel sour grapes-type pattern in stress
- Have a CDL of the lowest level of formal language complexity – no guarantee of a typology of matching complexity
- Constricting the constraint space in OT is not generally a viable strategy to avoid overgeneration
- Couched in stress but property of OT grammars in general – potentially true of *any* SL OT grammar

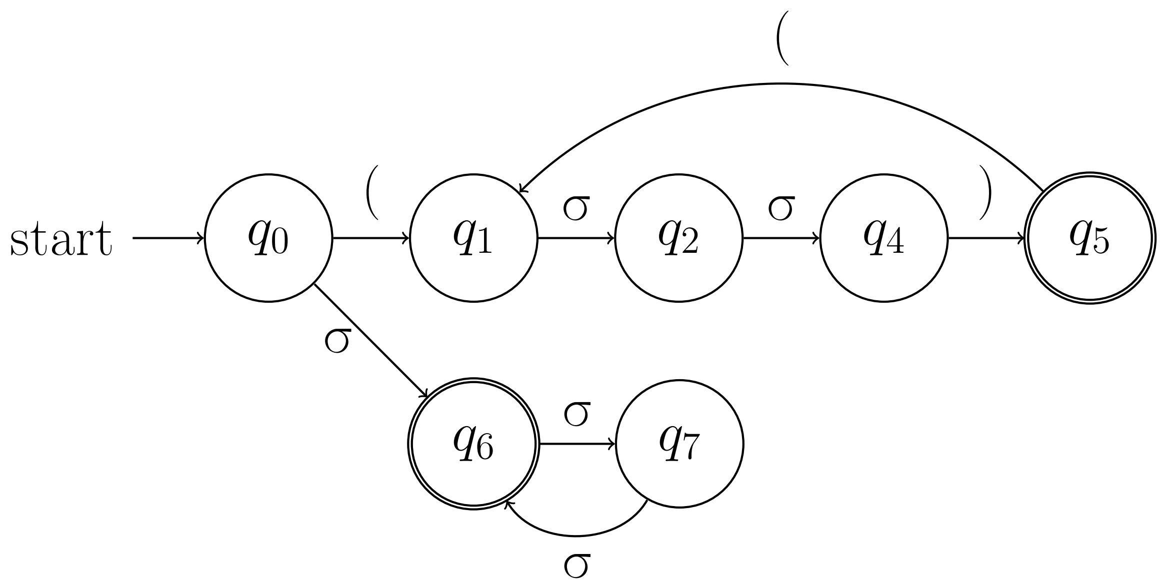
Future Work

- What happens with strictly piecewise constraints? Still CNL logic but adds precedence (non-local)
  - ALIGN-type constraints? Is e.g. ALIGN(F,R,Pwd,R,σ) writeable as SP constraint $\neg$)...σ...$]_\omega$ and does this produce things like the *Midpoint Pathology* (Eisner 1997; Hyde 2012)
- What is the typology of CDLs with other levels of logic?

Thanks!

Thanks to the audiences at NECPhon 2018, PhonX (Rutgers phonology reading group) and Bruce Tesar.

Appendix: Regularity



- Top path – only accepting state after a binary foot has been read
- Bottom path – only accepting state after an odd number of syllables and no foot boundaries have been read

Appendix: Not Star Free, Even n

Even n, $i = 1$, $v = \sigma$

$$
\begin{array}{lcccc}
 & uv^nw \in L & \rightarrow & uv^{n+i}w \in L \\
n & & & \\
2 & \sigma\sigma\sigma\sigma\sigma & & \sigma\sigma\sigma\sigma\sigma\sigma \notin L \\
4 & \sigma\sigma\sigma\sigma\sigma\sigma\sigma & & \sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma \notin L \\
6 & \sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma & & \sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma\sigma \notin L \\
 & & \vdots &
\end{array}
$$

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