

Overview

- Computational characterizations of phonology lead to restrictive, testable, and learnable theories of phonological processes (Heinz, 2018).
- Research question:** what kind of maps are **tone mapping patterns**?

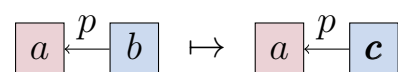
mo e rɛ ka ŋge ri e → mo e rɛ ka ŋge ri e
 L H L H L H L H
 (Kikuyu; Clements and Ford 1979)

- Result:** Maps defined with **quantifier-free least fixed point logic** give a restrictive, **output-local** characterization of tone mapping patterns

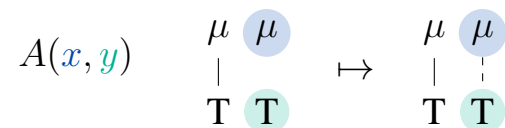
Logical maps

- Logical formulas define outputs through properties of the input (Courcelle, 1994)

$$c(x) \stackrel{\text{def}}{=} b(x) \wedge a(p(x))$$



- Tone mapping defines **association**

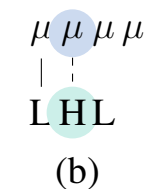
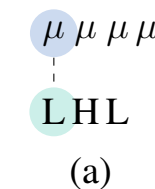
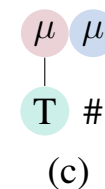
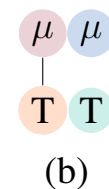


- These definitions
 - are **quantifier free** (Chandlee and Lindell, forthcoming)
 - use **least fixed point operators** (Libkin 2004), which allow recursive definitions (shown here with *implicit definitions*; Rogers 1997)
 - Use *either* predecessor (p) or successor (s)

Analyses

Mende (Left-to-right)

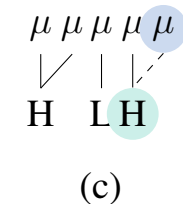
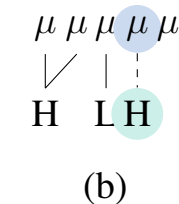
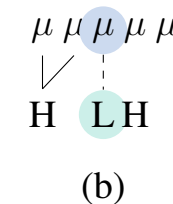
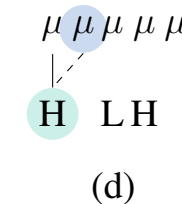
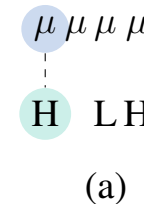
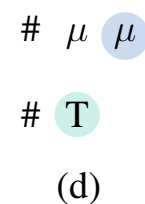
$$R(x, y) \stackrel{\text{d}}{=} \underbrace{(first(x) \wedge first(y))}_{\# \mu} \vee \underbrace{(R(p(x), p(y)))}_{\mu \mu} \vee \underbrace{(last(y) \wedge R(p(x), y))}_{\mu \mu}$$



$$A(x, y) \stackrel{\text{d}}{=} R(x, y)$$

Kikuyu (1st tone to 1st and 2nd TBUs; then left-to-right)

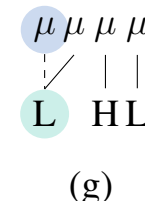
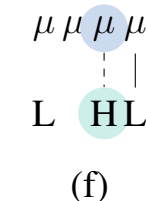
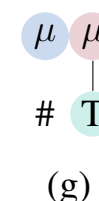
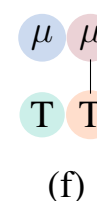
$$R(x, y) \stackrel{\text{d}}{=} (b) \vee (c) \vee \underbrace{(first(y) \wedge second(x))}_{\# \mu \mu}$$



$$A(x, y) \stackrel{\text{d}}{=} (a) \vee R(x, y)$$

Hausa (Right-to-left)

$$R(x, y) \stackrel{\text{d}}{=} \underbrace{(last(x) \wedge last(y))}_{\mu \#} \vee \underbrace{(R(s(x), s(y)))}_{\mu \mu} \vee \underbrace{(first(y) \wedge R(s(x), y))}_{\mu \mu}$$

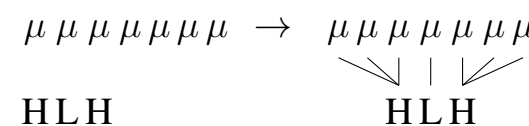


$$A(x, y) \stackrel{\text{d}}{=} R(x, y)$$

Discussion

- Begins solution for problem of logical complexity of tone mapping Jardine (2017)
- A principled characterization of the range of possible tone association patterns
- Can capture patterns that cannot be captured by OT ALIGN constraints

- Explains absence of unattested patterns, like centering:



- Recursive definitions provide the first logical definition of output-based locality for phonology

Select References

Chandlee, Jane and Lindell, Steven (forthcoming). A logical characterization of strictly local functions. In Heinz, Jeffrey, editor, *Doing Computational Phonology*. OUP.

Courcelle, Bruno (1994). Monadic second-order definable graph transductions: a survey. *Theoretical Computer Science*, 126:53–75.

Heinz, Jeffrey (2018). The computational nature of phonological generalizations. In Hyman, Larry and Plank, Frans, editors, *Phonological Typology, Phonetics and Phonology*, chapter 5, pages 126–195. De Gruyter Mouton.

Jardine, Adam (2017). On the logical complexity of autosegmental representations. In Kanazawa, Makoto, de Groot, Philippe, and Sadzadeh, Mehrmoosh, editors, *Proceedings of the 15th Meeting on the Mathematics of Language*, pages 22–35. London, UK. Association for Computational Linguistics.

Rogers, James (1997). Strict l12 : Regular :: Local : Recognizable. In Retoré, Christian, editor, *Logical Aspects of Computational Linguistics: First International Conference, LACL '96 Nancy, France, September 23–25, 1996 Selected Papers*, pages 366–385. Springer Berlin Heidelberg, Berlin, Heidelberg.

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