Tone association and output locality in non-linear structures

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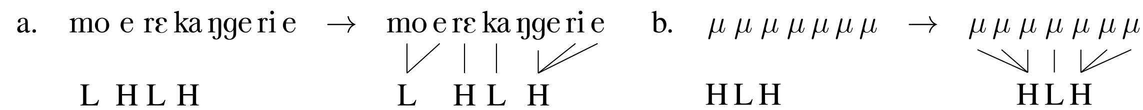
In this paper, we pursue a computational theory that distinguishes between possible and impossible autosegmental *tone mapping* patterns (Leben 1973; Williams 1976; Goldsmith 1976), in which unassociated tones are mapped to tone-bearing units (TBUs). A classic example is in Kikuyu (Clements and Ford, 1979), in which the first tone associates to the first two TBUs, and the remaining tones associate left-to-right thereafter (1a). An example unattested pattern is given in (1b), in which the melody and TBU tier align at their centers, with association proceeding outward.

(1)


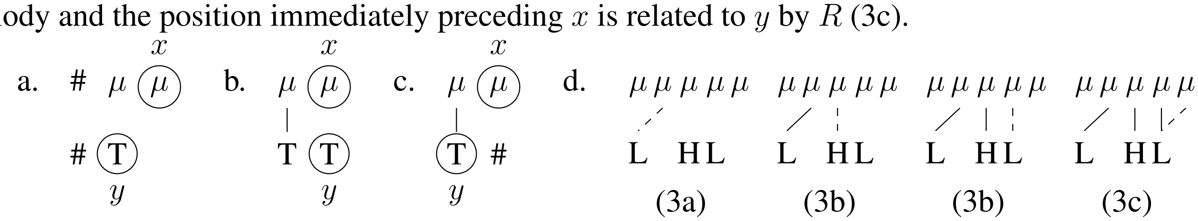
A computational characterization distinguishes attested and unattested tone mapping patterns by asking: what kind of functions are tone mapping patterns? The answer leads to restrictive, testable, and learnable theories of phonological processes (Heinz, 2018). While tone mapping patterns have been studied computationally as well-formedness conditions (Jardine, 2017a), there is no positive characterization of them as processes. Here, we show that tone mapping patterns are describable by a restricted *least fixed point logic*, which gives a typological characterization of the nature of tone association that is unavailable to derivation-based frameworks (as noted by Zoll 2003) but can capture patterns that cannot be captured by the alignment constraints used in optimization-based frameworks. It also solves a problem noted in Jardine (2017b) that tone mapping is beyond established computational characterizations for phonology. Finally, it also provides the first computational definition of output-based locality in functions for non-linear structures.

The output of a function can be defined through logical formulas relating elements in its input structure (Courcelle, 1994). For example, the Kikuyu tone mapping process in (1) asks: what underlying tone-TBU pairs should be associated in the output? We show that the range of tone mapping patterns can be defined with predicates that are *quantifier-free*, meaning that they identify *local* structures (Chandlee and Lindell, forthcoming), and that use *least fixed point operators* (Libkin 2004), which identify local structures in the *output*. (We use an abbreviated notation here.)

An example for Kikuyu is given in (2). Here, x and y are variables that range over positions in the autosegemental representation, and $p(x)$ indicates the immediate predecessor of x.

(2) $$R(x,y) \overset{\mathrm{d}}{=} \underbrace{\big(\texttt{second}(x) \wedge \texttt{first}(y)\big)}_{\text{(3a)}} \vee \underbrace{\big(R(p(x),p(y))\big)}_{\text{(3b)}} \vee \underbrace{\big(\texttt{final}(y) \wedge R(p(x),y)\big)}_{\text{(3c)}}$$

The formula $R(x,y)$ in (2) is interpreted as follows. Two elements x and y are related by R when either: x is the second position in the melody and y is the first position on the timing tier (depicted in (3a)); the positions immediately preceding x and y are related by R (3b); y is the last tone in the melody and the position immediately preceding x is related to y by R (3c).

(3)


The recursive interpretation of R is illustrated step-by-step in (3d). The complete definition of association in Kikuyu, including association of the first member of each tier, is given below in (4).

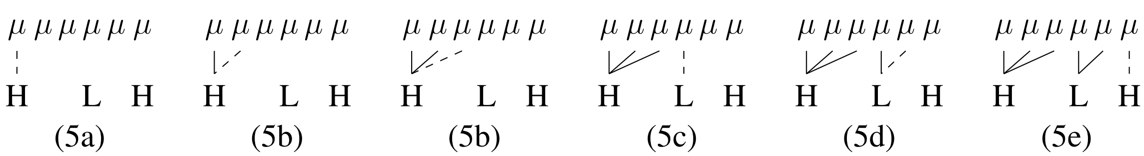
(4) $$A(x,y) \overset{\mathrm{d}}{=} \big(\texttt{first}(x) \wedge \texttt{first}(y)\big) \vee R(x,y)$$

We stipulate that these formulas can either refer to the predecessor p of an element, or its successor s (for right-to-left association patterns), but never both. (We do allow for reference to metrically strong positions on a tier: first, last, penult, etc.) Association of two elements is thus based only on *local* information within some window preceding (or following) the elements. Thus, while these formulas are language-specific, they form a restrictive class of functions, and cannot, for example, generate the unattested centering pattern in (1b). Furthermore, they align with Jardine (2017a)'s characterization of tone association as fundamentally local. We show that these formulas can capture directional patterns such as Kikuyu, Mende (Leben 1973) and Hausa (Newman 1986, 2000) and patterns that avoid H-spreading as in Kukuya (Hyman 1987, Zoll 2003).

To give another example, in the non-assertive tense in Northern Karanga Shona (Hewitt and Prince, 1989) two H tones associate in an 'edge-in' pattern—the first H associates to the first three TBUs and the second H associates to the final TBU. A L tone fills in the intervening TBUs. This is exemplified schematically in (6). A least fixed point formula for the pattern is in (5).

(5) $$R(x,y) \stackrel{\text{d}}{=} \underbrace{\big(\mathtt{first}(x) \wedge \mathtt{first}(y)\big)}_{\text{(a)}} \vee \underbrace{\big(\mathtt{first}(x) \wedge \neg\mathtt{fourth}(y) \wedge \neg\mathtt{penult}(y) \wedge R(x, p(y))\big)}_{\text{(b)}} \vee$$
$$\underbrace{\big(\mathtt{second}(x) \wedge R(p(x), p(y))\big)}_{(c)} \vee \underbrace{\big(\mathtt{second}(x) \wedge \neg\mathtt{last}(y) \wedge R(x, p(y))\big)}_{(d)} \vee \underbrace{\big(\mathtt{last}(x) \wedge \mathtt{last}(y)\big)}_{(e)}$$

The formula (5) works as follows: the first two elements on each tier are associated (5a); the first tone spreads up until the third TBU (avoiding the penult, so the second and third tones can associate; 5b); the second tone associates to the following TBU (5c) and the remaining TBUs up to the final TBU (5d); and the final tone and final TBU associate (5e). An example is given in (6).

(6)


The hypothesis that tone mapping patterns are least-fixed point, quantifier free functions compares well to other approaches. In derivational frameworks, association conventions are stipulated as left-to-right, right-to-left, or edge-in (Leben 1973, Godlsmith 1976, Hewitt and Prince 1989)—there is no explanation why a 'center-outward' convention is not possible. In OT treatments (Yip 2002; Zoll 2003), there is no explanation for why some ALIGN constraints are in CON and not others, such as the logically possible ALIGN constraints that can produce centering patterns (Eisner 1997). Furthermore, standard OT treatments cannot capture N. Karanga Shona (Zoll, 2003).

In contrast, the least fixed point characterization put forth here captures a wide range of patterns. It also explains why centering patterns are not attested, because they cannot be defined through local structures in the output. Future work can connect this notion of output locality in non-linear structures to learnability and to segmental processes (cf. Chandlee et al., 2015).

Selected references: • Chandlee, J. and Lindell, S. (forthcoming). A logical characterization of strictly local functions. In Heinz, J., editor, *Doing Computational Phonology*. OUP. • Clements, G. N. and Ford, K. C. (1979). Kikuyu tone shift and its synchronic consequences. *LI*, 10. • Heinz, J. (2018). The computational nature of phonological generalizations. In Hyman, L. and Plank, F., editors, *Phonological Typology*, pages 126–195. • Hewitt, M. and Prince, A. (1989). OCP, locality, and linking: the N. Karanga verb. In *WCCFL 8*. • Jardine, A. (2017a). The local nature of tone-association patterns. *Phonology*, 34. • Jardine, A. (2017b). On the logical complexity of autosegmental representations. In *MoL 15*. • Zoll, C. (2003). Optimal tone mapping. *LI*, 34.